



# Lecture10 (optics) two-mode squeezing and GRA experiment

two-mode squeezing  
GRA experiment

①  
 $\omega_p$  (two-mode squeezing - non-degenerate)  
 $\omega_a$  (two-mode squeezing - degenerate)  $|n_a, n_b\rangle$   
 $\omega_b$  (single-mode squeezing)  $|n\rangle$

Single-mode squeezing:  $\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + i\hbar\frac{\chi}{2}(\xi\hat{a}^\dagger - \xi^*\hat{a})$

two-mode squeezing:  $\hat{H} = \hbar\omega_1(\hat{a}_1^\dagger\hat{a}_1 + \frac{1}{2}) + \hbar\omega_2(\hat{a}_2^\dagger\hat{a}_2 + \frac{1}{2}) + i\hbar(\xi\hat{a}_1\hat{a}_2 - \xi^*\hat{a}_1^\dagger\hat{a}_2^\dagger)$

②  $\hat{H}_{int} = i\hbar(\xi\hat{a}_1\hat{a}_2 - \xi^*\hat{a}_1^\dagger\hat{a}_2^\dagger)$ ,  $\xi = re^{i\phi}$

$|\xi\rangle = e^{\xi\hat{a}_1\hat{a}_2 - \xi^*\hat{a}_1^\dagger\hat{a}_2^\dagger}|0,0\rangle = \sum_{n,m} C_{n,m}|n,m\rangle = \hat{1}|0,0\rangle = |0,0\rangle$

degenerate:  $\xi = r$   
 non-degenerate:  $\xi = re^{i\phi}$

$e^{\xi\hat{a}_1\hat{a}_2 - \xi^*\hat{a}_1^\dagger\hat{a}_2^\dagger} = 1 + \xi\hat{a}_1\hat{a}_2 + \frac{\xi^2}{2!}\hat{a}_1^2\hat{a}_2^2 + \dots$

$|0,0\rangle = |0,0\rangle$   
 $+ (\xi\hat{a}_1\hat{a}_2 - \xi^*\hat{a}_1^\dagger\hat{a}_2^\dagger)|0,0\rangle = |1,1\rangle$   
 $+ \frac{1}{2!}(\xi\hat{a}_1\hat{a}_2 - \xi^*\hat{a}_1^\dagger\hat{a}_2^\dagger)^2|0,0\rangle = |2,2\rangle + |1,1\rangle|0,0\rangle$   
 $+ \frac{1}{3!}(\xi\hat{a}_1\hat{a}_2 - \xi^*\hat{a}_1^\dagger\hat{a}_2^\dagger)^3|0,0\rangle = |3,3\rangle + |2,2\rangle|1,1\rangle + |1,1\rangle|0,0\rangle$   
 $+ \dots$

$(\hat{a}_1\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_2^\dagger)(\hat{a}_1\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_2^\dagger) = \hat{a}_1^\dagger\hat{a}_2^\dagger = \hat{a}_1^\dagger\hat{a}_2^\dagger$





✓ two-mode squeezing

- GRA experiment

$$|\xi\rangle_2 = \hat{S}_2(\xi)|0,0\rangle = \sum_{n=0}^{\infty} \frac{(-1)^n}{\cosh r} \frac{(\tanh r)^n}{n!} |n,n\rangle, \quad \xi = re^{i\theta}$$

$$\begin{cases} P_n^{(1)} = P_n^{(2)} = \frac{(\tanh^2 r)^{2n}}{\cosh^2 r} \\ \bar{n}_1 = \bar{n}_2 = \sinh^2 r = \bar{n} \end{cases}$$

$$\rightarrow P_n^{(1)} = P_n^{(2)} = \frac{(\sinh r)^{2n}}{(\cosh r)^{2n+2}} = \frac{(\sinh^2 r)^n}{(1 + \sinh^2 r)^{n+1}} = \frac{\bar{n}^n}{(1 + \bar{n})^{n+1}}$$

thermal light photon distribution

$$P_n = \frac{e^{-\bar{n}} \bar{n}^n}{n!} = \frac{\bar{n}^n}{(1 + \bar{n})^{n+1}}$$

$$\frac{1}{1 + \bar{n}} = e^{-\frac{1}{\bar{n}}}$$

$$r \uparrow \rightarrow T_{eff} = \frac{\hbar\omega}{2k_B \ln(\cosh(2r))}$$

Grangier - Roger - Aspect Experiment (GRA)



